Computational Neuroscience

taken as class notes for the Fall 2024 class with Prof. Wen

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Contents

Chapter 1 Candid Lecture Notes

Scripted with the aid of ChatGPT, me taking notes during class, TAs notes, and the official lecture notes.

1.1 Sep 9 - Neuroelectronics of a single neuron

- Basic electrical properties of neurons.
- Mathematical models describing neuronal dynamics.
- Simple neuron models, such as the integrate-and-fire model.
- Detailed models like the Hodgkin-Huxley model, which includes multiple voltage-dependent conductances.

Membrane Potential, Capacitance, and Resistance

- Neurons contain a variety of ions, molecules, and proteins.
- Ion channels in the cell membrane are often highly selective, allowing specific ions to pass.
- The permeability of ions across the membrane, combined with their concentration gradients, determines the membrane potential.

By convention, the potential outside the cell is set to 0. The membrane potential V inside the neuron is typically negative due to the diffusion of K^+ ions out of the cell, leaving behind excess negative charge.

Scale of Membrane Potential

The membrane potential is influenced by thermal energy and should balance the needs of electrical signaling and thermal fluctuations. The thermal energy of a single ion is given by:

$$k_B T$$

where k_B is the Boltzmann constant. The energy gained or lost by an ion traversing the membrane is:

$$qV \sim k_B T$$

Substituting values, $k_B = 8.6 \times 10^{-5}$ eV/K and $T = 300$ K, gives:

$$V \sim 26 \text{ mV}$$

Experimentally, membrane potentials range between $+50$ mV to -80 mV, or $+2$ to -3 times the estimated voltage.

Membrane Capacitance and Resistance

- Membrane capacitance C_m is proportional to the surface area of the neuron.
- Specific membrane capacitance c_m is approximately 10 nF/mm^2.
- Membrane resistance R_m is inversely proportional to surface area, with specific resistance $r_m \approx 1$ MΩ mm^2.

The membrane time constant $\tau_m = R_m C_m = r_m c_m$ is independent of the total membrane area and typically falls between 10-100 ms.

Nernst Equation

The membrane potential at resting state can be derived using the Nernst equation, considering a single type of ion (e.g., K^+):

$$E = \frac{k_B T}{ze} \ln\left(\frac{n_{\text{out}}}{n_{\text{in}}}\right)$$

This potential is also known as the reversal potential E_K. For K^+, E_K typically ranges from -70 to -90 mV.

Goldman-Hodgkin-Katz (GHK) Equation

For channels that are not selective, the Goldman-Hodgkin-Katz equation provides the reversal potential:

$$E_m = \frac{k_B T}{e} \ln\left(\frac{\sum_{i=1}^{N} P_{M_i^+}[M_i^+]_{\text{out}} + \sum_{i=1}^{N} P_{A_j^-}[A_j^-]_{\text{in}}}{\sum_{i=1}^{N} P_{M_i^+}[M_i^+]_{\text{in}} + \sum_{i=1}^{N} P_{A_j^-}[A_j^-]_{\text{out}}}\right)$$

where P represents the permeability of each ion type.

Sodium Anomaly

The equilibrium potential for sodium E_{Na} is approximately $+50$ mV, much higher than the resting potential $\Delta V = -60$ mV. This discrepancy is due to the sodium anomaly, where sodium ions have limited permeability under typical experimental conditions.

Ion Pumping Mechanism

Cells maintain a sodium-potassium balance through active ion pumping. The sodium-potassium pump continuously hydrolyzes ATP to pump Na^+ out and K^+ into the cell, maintaining concentration gradients essential for neuronal function.

Membrane Current

The total current across the membrane is described by:

$$I_m = \sum_i g_i (V - E_i)$$

where g_i are the conductances of different ion channels, and E_i are their reversal potentials.

For leaky channels and ion pumps, the membrane current I_L can be described by:

$$I_L = \bar{g}_L (V - E_L)$$

Combining all contributions:

$$C_m \frac{dV}{dt} = -\sum_i g_i (V - E_i) - \bar{g}_L (V - E_L) + I_e$$

where I_e is an external current.